# A review of structural size optimization techniques applied in the engineering design.

ErsilioTushaj, Niko Lako

**Abstract** -The efficient solution that satisfies optimality condition is an important issue when analyzing the structural engineering design problem. The new codes of structural design consist in methodologies that demand the total exploitation of the resources of the construction materials. The goal of an optimal design is to achieve the best feasible solution according to a measure of effectiveness. This demand, emphasizes the need for lower weights and total costs of the realization of the structure. The first analytical work in structural optimization was by Maxwell in 1890, followed by Mitchell in 1904. The best applications during the years 30'-50's were in the aircraft industry, consisting of compressive loads and buckling constraints. Good developments followed with Schmit in 1960. He was the first to offer a comprehensible analytical statement of the problem. Up to now have been published a lot of algorithms, which have similar approaches but offer different results. It is difficult to identify which one is appropriate in solving a specific structural design problem. For this purpose, more studies, surveys and analysis are necessary to give developments, advantages and disadvantages in the application of these algorithms. The goal of this paper is to analyze and follow an up to date study of size optimization techniques used in the structural optimization design, priory in steel constructions, basing on some highly peer reviewed studies, that were possible to be analyzed by the author.

**Index Terms** - review, structural optimization, size optimization, engineering design.

## 1. INTRODUCTION

The design process in structural optimization as announced by professor Uri Kirsch [1] in 80's, can be classified into four stages: formulation of functional requirements, the conceptual design stage, optimization and detailing. Iterative procedures or the application of algorithms are necessary before the final solution is achieved.

Typically, an optimal design problem consist of three issues: (a) objective function, (b) design variables, and (c) constraints. Usually in the optimization problem the objective function used is the total weight or the cost of the structure. The design variables are those parameters to be determined by the designer in order to generate an optimal solution. In practical applications achieving an optimum design should be carried out with respect to a set of strength, stability and serviceability limitations. The classification of the structural optimization problems related to the type of design variables involved, is divided into three main categories: sizing, shape, and topology optimization. In sizing optimization the cross sectional areas of the structural members are considered as design variables. The process of sizing can be continuous or discrete. In a continuous sizing optimization any positive value can be assigned to the cross sectional areas of the elements.In practical cases the structural members should be adopted from a set of available sections, so the design problem turns into a discrete sizing optimization. [2]

More efficient and robust algorithms are necessary to respond to the actual demand of design. Optimization problems may fall into the trap of local minima, or may require too much iterations to guarantee the optimal solution. Efficiency and robustness of the algorithms are the two bigger goals of researchers. [3]. Structural optimization is mostly not taught in the bachelor or master programs for civil engineers, but it is considered as a complementary course in some PhD programs; such as the course followed by professor Bontiempi of structural analysis at the University "La Sapienza", in Rome. [4]

### 1.1. First steps in structural optimization.

The first analytical works on structural optimization can be dated back to 1890 by Maxwell, followed by other studies of Mitchell in 1904. The structural optimization techniques were next developed in the aircraft and space industry, as a consequence of restrictive requirements on the problem of the minimum weight design problem in the engineering design. During the years 30-50', the availability of computers, made possible the application of linear programming techniques to the plastic design of frames.

### 1.2. Structural optimization after the 60's.

Lucien A. Schmit[5], was the first to offer a good statement on the use of mathematical programming techniques in solving the nonlinear inequality constrained problem of designing elastic structures, when applying a multiplicity of loading conditions. This work introduced a new philosophy of engineering design which only in the 80's began to be broadly applied. It indicated the feasibility of coupling finite element structural analysis and nonlinear mathematical programming to create automated optimum design capabilities. In the 60's, the computational experience indicated that mathematical programming (MP) techniques, applied to structural design, were limited to only a few of dozen of design variables. At the beginning the applications were limited to relatively small structures. In the late 1960's an alternative approach, called Optimality Criteria (OC), was presented in analytical form.

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The (OC) was largely intuitive and it was shown to be a very effective design tool. It was independent of the problem size and usually provided a near optimum design with a few structural analyses. This feature represented a remarkable improvement over the number of analysis required for (MP) methods to reach the solution. The (OC) optimality criteria method was presented in two different approaches, as a physical or a mathematical tool. The physical problem was based on stress constraints (fully stressed design – FSD) or displacement one (Fully displacement design – FDD). An integrate version also came regarding both stresses and displacements constraints in "Fully Utility Design" (FUD. The mathematical approach was based on the Kuhn-Tucker conditions.

### 1.3. Advances in structural optimization.

Other optimization techniques have emerged in the last decades. These methods do not require gradient information for the objective and the constraint functions, but use probabilistic transition rules rather than deterministic ones. These techniques are called stochastic or meta-heuristic approaches, since they search the optimal solution generating random populations, based on some criteria or fitness functions. The idea behind them is to simulate a natural phenomenon, such as survival of the fittest, the immune system, swarm intelligence or the cooling process of molten metal through annealing. A detailed review of these algorithms as well as a comparison of their performance for discrete sizing problems is provided by Hasancebi[6]. These heuristic optimization methods have some advantages when compared to the deterministic one. They separate the domain knowledge from search, making them generally applicable to a wide variety of problem formulations; with no limitation on the continuity of the search space, since no gradient information is required.

The state of art of Kazemzadeh[7], makes a distinction of the methods in traditional and modern one. The traditional methods include the mathematical programming (MP) techniques and the optimality criteria (OC). The modern methods include the non-deterministic approaches. Some of the most recognized modern algorithms are: the genetic algorithms (GA), harmony search method (HS), simulated annealing (SA), particle swarm optimization (PSO), ant colony optimization (ACO), ant bee colony (ABC) etc.

The meta-heuristics also present some disadvantages. They require significantly more computational resources than the deterministic one. Research on convergence has shown that the number of evaluations required to reach a given solution, grows as a function of the square root of the size of the problem. The slow rate of convergence towards the optimum and the need of the high number of structural analyses are still conceived as the main shortcomings of these techniques. A possible solution is by taking advantage of computing methods, such as parallel or distributed methods of workload amongst multiprocessors, which are connected to each other.

## 2. Generals in optimization techniques.

### 2.1. A General Formulation of the Optimization Problem.

The general nonlinear constrained optimization problem can be stated as followed by [8]:

$$\begin{gathered} find \quad \underline{x} \\ minimize \ \ f(\underline{x}) \\ subject\ to\ g(\underline{x}) = 0 \ \ and \ h(\underline{x}) \geq 0 \end{gathered}$$

where: $\underline{x} = (x_1, x_2, \dots, x_n)$ is a set of variables, which can be binary, discrete or continuous.

$f(\underline{x})$is the objective function.

The goal of the optimization process is to minimize the value of $f$ related to $\underline{x}$.

### 2.2. Different levels of structural optimization.

Many authors agree on the following classification about the different levels of optimization:

- *size optimization* deals with minimization (or maximization) of one or more response variables (such as stresses, deformation, stiffness) acting on one or more design variables;
- *shape optimization* aims to find the optimal shape of domain, which is no longer fixed and become a design variable itself;
- *topology optimization* for continuum structures deals with the number, position, shape of holes and topology of the domain.

### 2.3. Trends in structural optimization.

In the process of solving these problems, it is important to be attentive to the objective function, because the right selection of it can bring to better results. The common objective functions used, as documented by the literature, are the following: minimization of weight, maximization of stiffness, minimization of cost design and any combination of the previous. The minimization of weight is the most used, and often there is an implicit assumption that the weight of a structure is the best measure for evaluating its cost. But this is not true, since doing this, there is a risk of neglecting other important terms, beside material cost, that contribute to define the final value. A recent modern point of view by some critics, is that the objective function should be evaluated in terms of the life-cycle cost of the structure, taking in account more terms, such as the costs of materials, fabrication, erection, maintenance, disassembling at the end of structure life etc. So it's clear that this approach, is more realistic, but encounters additional difficulties. Unfortunately, constraint evaluation in the real world, involves many sources of imprecision and approximations that reflects into the final results.

Some authors agree that during the architectural-engineering design process, the optimal version should not be the first goal


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of the design team. It is necessary to explore various possible optimal solutions, which can also offer better quality from an aesthetic point of view.[9]

### 2.4. The algorithmic approach.

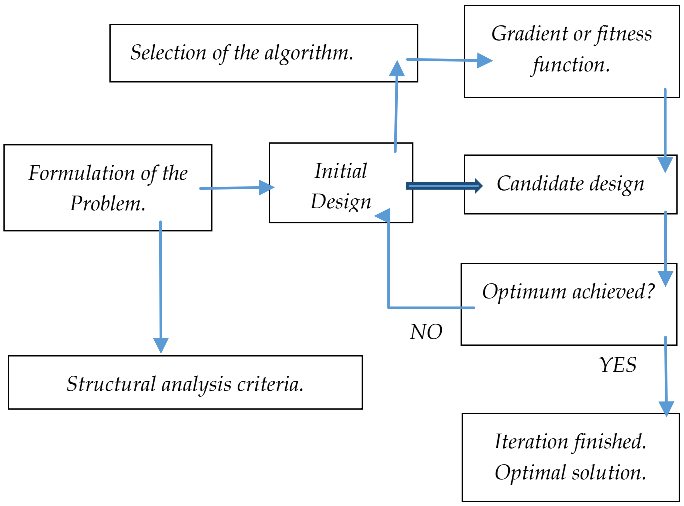


**Fig. 1.** A general flowchart of the optimization problem.

The variables, objective function and the constraints represent physical properties of the structure being optimized. The algorithms deal exclusively with the mathematical form of the problem interfaced with computer models representing the physical structure. The model is used to perform structural analyses requested by the optimization algorithm. The interaction scheme between the algorithm and the analysis tool depends on the optimization method. *Fig. 1*shows a general scheme of a gradient based algorithm. The algorithm generates a design by assigning values to the optimization variables. After being updated with new values, the structural model is used to perform analysis. The results are taken into account by the algorithm to generate new designs, as long as the termination criteria are not met. Optimization time is reduced by choosing an algorithm that converges quickly towards the optimal design.

### 2.5. Difficulties in practical applications.

The computational methods have been successful in many fields of engineering in the recent decades. The finite element method, for example has proven to be of common use not only in academic contexts but also between professional engineers. Finite element applications are now essential tools for modern design. The advances in technology bring to us computers much more powerful than the ones that existed before. The same thing cannot be said also for numerical methods optimization methods, which despite their advance, are limited in the academic research or are exclusive to specialized companies. Practical applications are rare. These are encountered much more in mechanics, aeronautics and electronics.

Their rarity is firstly due to the fact that the subject is really complex, requiring a deep knowledge and solid background in many topics of numerical methods and also structural mechanics. Secondly there does not exist a unique formulation of an optimization method that can be successfully applied to a really large class of problems. The creation of a "multipurpose" software that could manage sufficiently a wide set of structural problems has not been possible yet. Baldock[10] has reported a short review of some real world applications of structural optimization and softs that have been used for this purpose.

Algorithms are programmed in softs. Some of the most used are BASIC, MATLAB, FORTRAN, C++ etc. Other ad hoc softs have been developed to respond to practical applications such as: TOSCA2 (FE-Design), GENESIS3, OptiStruct4, BIGDOT, ABAQUS, OPTIMA, SODA, and others. A review of these soft has been done by Vanderplaats[11]. Abishek[12] has documented MATLAB codes to be used in most known optimization problems.

## 3. The Optimization Algorithms.

This paragraph explains through a short introduction how the optimization algorithms have evolved. A review for the discrete variable optimization is given by Arora[13]. A review of non-deterministic approaches with algorithmic steps was done by Hare [14].

### 3.1. Deterministic methods.

The complexity of applying structural optimization algorithm to practical problems has always motivated the researchers to develop more efficient and robust optimization techniques. A review of applications of deterministic methods for structural optimization has been studied by Chain [15].

#### 3.1.1. Mathematical Programming (MP).

Mathematical programming techniques are among the most known classes of optimization techniques. The (MP) optimization problem is divided into: the unconstrained and the constrained (MP) problem. Since material, strength and displacements in structures are limited, consequently the problem is generally in the constrained form. The general nonlinear formulation of the constrained problem was given in 2.1. The linear formulation, (LP) linear programming, can be solved using graphical or simplex methods. The solution obtained is possible, only when a space with few variables is analyzed, since calculations get difficult in correspondence with the complexity of the problem [16]. Other solution methods have been developed, based on gradient analysis of the objective function. The basic idea is to move in the negative direction of the gradient of the objective function to find a more promising candidate design. Some of the gradient based methods are: the

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"Conjugate gradient method" by Fleetcher[17], the method of the feasible direction [18], the Sequential linear programming by Kelley[19] etc.
It is generally believed that (MP) techniques are not robust methods for optimum design of structural systems that analyze numerous design variables. They can fall into the trap of local minima if some conditions are not imposed in the right way.

*Sequential Linear Programming (SLP) (1961)*
The Sequential Linear Programming (SLP) with move limits, introduced first by Stewart [20], has been a successful tool in size optimization of steel truss structures by Stasa[21]. A program called (OPT-KM) has been built to find the optimal solution of the minimal weight with stress and displacement constraints. The (OPT-KM), built in BASIC language, has been compared to the (KM) algorithmic program using the (FSD) – "Fully Stressed Design" of the Optimality Criteria (OC). Advantages of the (SLP) are reported when approaching with different move limits. (See also 3.1.2).

*Gradient Based Algorithm (2005)*
In 2005, Gisbain[22] reported a gradient based algorithm, in MATLAB environment that applied to the optimization of steel structures. The algorithm applies minimization of the stiffness matrix of the truss structure. The process combined size and topology optimization.

#### 3.1.2. Optimality Criteria (OC).

Another class of non-deterministic structural optimization techniques covers the optimality criteria (OC) methods. In order to generate an optimum design, a recursive algorithm is employed to update the structural members for satisfying the (OC). Early works on the algorithm are due to Prager[23] and Venkayya[24]. Then Bazaraa[25] realized the necessary optimality criteria in Mathematical Programming in the presence of differentiability. So it was possible to develop another version of (OC), based on the Kuhn-Tucker conditions, and Virtual Work in searching the optimal solution [26].
Later, numerous variants of the optimality criteria methods have been applied to optimum design of pin-jointed and frame structures. It is worth mentioning that the fully stressed design (FSD) can be also considered as a simple stress-ratio optimality criteria technique which can only deal with stress constraints. An extension of the FSD that handles both stress and displacement constraints is the fully utilized design (FUD), which is capable of generating a feasible solution through a small number of structural analyses.
Stasa[21], comparing the (OC) algorithm in the "fully stressed design" version, with the (SLP) with move limits; has concluded some advantages of the (FSD). The (FSD) is more efficient, it needs less iterations, and in general if offers better optimal solutions, but is less effective with displacement constraints since it doesn't consider them. The advantages of the (SLP) with move limits are enhanced, when considering the right values of the move limits. (OC) algorithms have been also applied in the topology optimization. Results have shown difficulties in the first steps of applying the algorithm to the problem, but greater effectiveness in obtaining results.[27]

*Fully Constrained Design (FCD) (2014)*
In the last years the efficiency of traditional methods in finding the optimal solutions has inspired the scientific community, from the University of Stanford, ETH Zurich and Phoenix System to develop a new algorithm which can find the optimal solution through a similar process of the (OC). This algorithm has been called the (FCD) "Fully Constrained Design" [28]. It has been proved to be efficient and robust, in comparison to other deterministic and non-deterministic algorithms.

### 3.2. Non-deterministic techniques.

These techniques have been built based on other system phenomena and follow search strategies in finding the optimum design. There is a rising popularity of these techniques as a consequence of the fact that they don't require gradient information, thus they are independent from the population under analysis. They can deal with discrete and continuous design variables and can handle better problems with a high number of variables. They are not too difficult in coding, compared to deterministic methods. Some reviews on these methods have been done in the last years by Kazemzadeh[7], Kicinger[29], Lamberti[30], Hare [14], and others.
Basically the aim of these methods is the location of the global optimum, by generation of candidate solutions in an iterative way. The fundamental idea is to seek the vicinity of more promising candidate designs, found so far to drive the search towards more reliable solutions. It starts with an initial population randomly selected design. Then the other populations are generated in base of a rule. In order to investigate their quality, each candidate design is evaluated with respect to the objective function of the problem.
Once fitness of each candidate design is computed, new candidates can be generated using the obtained information from the formerly generated designs. The generation of a new population is guided by a mechanism or an operator and it is iteratively performed until a predefined termination criteria is obtained. The last iteration is expected to be the optimal or an acceptable near optimal result.

#### 3.2.1. Evolutionary algorithms (EA).

These type of methods use a so called evolutionary computation (EC), which uses computational models of processes of evolution and selection. These are mechanisms of Darwinian evolution and natural selection encoded in evolutionary algorithms. These algorithms are divided in: **a.** Evolution strategies (ES) (1965) by Rechenberg[31]; **b.** Evolution programming (EP) (1966) by Fogel[32], **c.** Genetic Algorithm (GA) (1975) by Hol-


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land [33], and **d.** Genetic Programing (GP) (1992) by Koza[34].

*Genetic Algorithms (GA) (1975)*

The most known of the (EA)-s, are the genetic algorithms (GA), which have found a wide spectrum of applications in diverse engineering disciplines. The (GA) uses concepts from evolutionary biology. A genetic algorithm with standard components referred to as simple GA is outlined as follow [35]:

*Step 1.Initial population:* A design variable is encoded as a binary string, such as 1001001. Initial population incorporates a predefined number of individuals.

*Step 2.Encoding:* For each individual a decoding is used to map all substrings to some integer values representing the sequence numbers of standard sections.

*Step 3. Evaluation and fitness.* Once an individual is decoded, analysis is done in order to obtain the structural response under external loads. Each individual is assigned with a fitness score, which indicates the merit of the individual with respect to the overall population.

*Step 4. Selection and reproduction.* A selection is done where individuals of high fitness scores are selected and reproduced.

*Step 5. Crossover.* The selected and reproduced individuals are paired, in base of a crossover probability. *Step 6. Mutation and termination*. Mutation is applied by randomly altering a gene of 0 to 1 with a probability of less than a given value. The new population replaces the old one and the steps are repeated until a number of iterations is reached.

Applications of the evolutionary algorithms have been reported widely in the scientific community. The above Simple (GA), has been improved several times leading to better results. (ES) applied to truss structures by Hasancabi[6], showed a minor weight of the structure at a minor time. The efficiency and the robustness of the (ES) has been proven by the study.

The four typologies of the evolutionary algorithms have the same structure, they try to mimic the evolutionary process in nature, and the Theory of Darwin. They change each one with the others in the way the process is realized and the fitness function is applied.

Some advantages of enhanced versions of the genetic algorithms (GA) combined with other deterministic algorithms, have been documented too [36]. Genetic algorithms gave good results in the layout optimization of truss structures, where topology, shape and size optimization is executed at the same time[37].

Sisko[38] developed a design center system based on decision system support for conceptual design at early stages of the construction project, considering architectural and structural aspects at the same time.

### 3.2.2. Swarm Intelligence Algorithms.

The most known algorithm in this category is the Particle swarm optimization (PSO).

*Particle Swarm Optimization (PSO) (1995)*

The PSO algorithm, proposed by Kennedy [39], is an important search technique with good applications in the field of structural design optimization. The (PSO) is based on the social behavior of animals, concerned with grouping by social forces that depend on both memory of each individual as well as the knowledge gained by the swarm. The particles through the search space and their positions are updated using the current positions, the velocity vector and a time step. A swarm consists of a predefined number of particles referred to as swarm size.

All the particles are analyzed with the values of design variables that they represent and their objective function values are calculated**.** A particle's best position is referred to as particle's best and is stored separately for each particle in a vector B. On the other hand, the best feasible position located by any particle since the beginning of the process is called the global best position, and it is stored in a vector G. The values are updated.The velocity vector of each particle is updated considering the particle's current position, the particle's best position and its global best position. The position vector of each particle is then updated with the updated velocity vector. Steps are repeated until a predefined number of iterations N.

*Ant Colony Optimization (1991)*

Ant colony optimization (ACO) technique is inspired from the way that ant colonies find the shortest route between the food source and their nest. The technique has been developed by Colorni and Dorigo[40]. Other publications with application of the improved algorithms, have reported good results too.

*Artificial bee colony algorithm (ABC) (2005)*

Another novel meta-heuristic algorithm is a nature-inspired method, the artificial bee colony (ABC). This algorithm was proposed in its original version by Karaboga[41]. Sonmez[42] has used a discrete ABC algorithm for optimum design of truss structures with 582 members and has reported a good performance of the algorithm compared to the others known meta-heuristic techniques.

*Enhanced Honey Bee Mating (EHBMO) (2017)*

The Honey bee mating is a swarm based algorithm, where the search algorithm is inspired by the process of mating in honeybees. The (EHBMO) is a very recent optimization algorithm, which applied an enhanced version of the original one. The algorithm is reported to be very competitive with other meta-heuristic methods analyzed. The (EHBMO) uses the concept of giving weight to distant candidates which are slightly less feasible than the current local candidate, but may hold information about the global optimal solution of the problem. The robustness of the algorithm has been proven by the study. [43]

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### 3.2.3. Physical related Algorithms

These algorithms are related to the imitation of physical phenomena, in processes that build an algorithm that finds the optimal solution of a problem.

*Simulated annealing (SA) (1985)*

The first publication on Simulated Annealing processes was done in 1985 by Cerny[44]. Simulated annealing (SA) extends its process to the annealing of physical systems applied in thermodynamics. A physical system initially at a high-energy state is cooled down gradually until its minimum energy level is reached. This process can be simulated to solve optimization problems. Some applications of this algorithm in steel structural optimization have demonstrated good results[45]. An efficient simulated annealing algorithm for design optimization of truss structures has been reported by Lamberti[46].

*Harmony search algorithm (HS) (2001)*

Harmony search is a non-deterministic method, inspired by the improvisation process of musicians, firstly proposed by Geem[47]. Further developments and applications have been reported of the (HS). A survey for this purpose was done by Manjarres[48].

The idea comes from the process of creation, where musicians make several harmonies until they find the desired one. It is the improvisation process, in which a musician tries to find the perfect harmony, examining a wide range of combinations. Although (HS) algorithm is a random search technique similar to the genetic algorithm and the particle swarm optimization, it is considerably different from those population based evolutionary methods due to its single evolving search memory. Geem has used the algorithm for sizing optimization of truss structures and proved to be more efficient than conventional mathematical methods and genetic algorithms. The algorithm can also be employed for optimum design of other types of structures such as frame, plate or shell structures. Shabani[49] introduced recently in 2017, a new version of the harmony search, the "Selective Refining Harmony Search" (SRHS).

*Big bang crunch optimization. (BB-BC) (2006)*

Erol[50] in 2006, introduced a new meta-heuristic optimization method called Big Bang–Big Crunch (BB-BC). The algorithm has been proven to be efficient in tackling practical optimization problems, and has become a popular meta-heuristic.

Some publications have reported the design optimization of planar and spatial truss structures, performed using a modified version of the algorithm. In order to increase the efficiency of the (BB–BC) algorithm, a weighting parameter has been introduced to control the influence of both the center of mass and the current global best solution on new candidate solutions. Further, a multiphase search strategy has been employed to increase the quality of the final solution. The efficiency of the BB–BC algorithm was compared to previously reported (GA), (PSO) and (ACO) based approaches.

*Charged system search (CSS) algorithm (2010)*

Charged system search (CSS) has been proposed by Kaveh and Talatahari in 2010 [51]. The algorithm has been employed for the solution for optimum design of skeletal structures including three trusses and two frame structures. The study has reported advantages of (CSS) in comparison to the other meta-heuristic methods. Some enhanced versions of the (CSS) have been applied for configuration optimization of truss structures[52].

### 3.2.4. Other stochastic algorithms.

Stochastic optimization methods are processes, where the search for the optimal solution is done by generating new populations that satisfies better the result, using some random variables.

*Guided Stochastic Search Algorithm (GSS) (2014)*

The GSS offer a design procedure, built ad hoc for steel trusses, where the generation of the optimal solution is guided by the virtual work and other criteria of responses of the algorithm. The information provided through the structural analysis and design check stages are utilized for handling strength constraints. The Virtual Work Theorem is applied to achieve the displacement constraints. The optimization with minimum weight is performed based on both strength and displacement criteria. The method has been reported to be efficient in truss structures compared to other algorithms, using from 10 to 349 sizing variables. (GSS) is a recent method proposed by Kazemzadeh and Hasancabi[53]

Stochastic search techniques perform random moves using strategies taken from nature to locate the optimal solution using a single or a population of candidate designs. Since the (GSS) takes the response computation from generated designs, it is possible to utilize such valuable information to the design process, obtaining better results, in a shorter time.

*Tabu search. (TS) (1989)*

The Tabu Search (TS) has been proposed by Glover[54] in 1989, and it works with other algorithms to overcome the local optimality. Its applications are reported mostly on discrete constrained combinatorial optimization problems. The tabu search uses a local neighborhood search procedure moving from a potential solution to another in the neighborhood, until some stopping criteria are satisfied. To avoid falling into a trap of local minima, the solutions admitted to the new neighborhood, are determined through the use of memory structures.

### 3.3. Comparison of structural size optimization algorithms.

In the previous paragraphs only some of the most peer reviewed algorithms have been introduced. The variety of these

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techniques, available in the literature of size structural optimization, make it difficult to choose the right method that is necessary to be applied in the practical applications.

Most of the studies analyzed, demonstrate that if the algorithm is improved from the original form, it can lead to better solutions in a shorter time of iterations. If the criteria are chosen in the right way, most of the recently developed algorithms reported good results.

The comparisons between the design optimizations for size truss structures, have shown the superiority of Simulated Annealing (SA), Evolution Strategies (ES), Enhanced Genetic Algorithms (GAs), Fully Constrained Design (FCD), Guided Stochastic Search (GSS), Enhanced Artificial Bee Mate Colony (EHBMO) etc.

### 3.4. Practical applications.

Some studies conclude that the structural optimization algorithms are efficient tools to solve real world problems. [55]. These have been mostly recorded in the automotive industry using topology optimization.

(OPTIMA) System was developed by HKUST [56]. It has been applied to the size optimization of some project for high buildings in Honk Kong, using combinations of (OC), (GA) and structural analysis. The "Kowloon Mega Tower", in China, was built optimizing the material costs and distances of the constructions. [36]

ODA (Structural Optimization Design Analysism acronym software Inc.), is another commercial software, realization of Waterloo University, which applied (OC) criteria to optimization. A collaboration of Ove Arup, Partners Honk Kong ltd and Khust brought to life the North-Eastern Tower of the Honk Kong Station, with objective function the minimal weight.

The Train Station roof of Florence, realized in 2002, with dimensions of 150m x 26m x 15m height, has been projected applying an Extended ESO algorithm [57].

## 4. Conclusions.

The surveys and the conclusions given in this study are based on the references analyzed. There is a widespread information, with too many studies about the argument, and it is not possible to analyze all them in one paper.

The structural optimization algorithms were divided in two maxi-groups: the deterministic and non-deterministic methods. The two bigger goals are: efficiency in the number of iterations and robustness in finding the optimal solution. The deterministic methods have the advantage of requiring less iterations to the optimal design; but have the disadvantage of managing only problems with less than 100 variables. Instead the non-deterministic methods, generate candidate designs using a fitness equation that is not influenced by the values of the candidate optimal design, so may handle bigger size problems, and guarantee a greater effectiveness. As disadvantage they need more iterations.

New algorithms are constantly being developed in the field of size structural optimization. These algorithms make it possible to have faster optimal results. These algorithms can handle problems with a higher number of variables. Some of the most peer reviewed algorithms have been analyzed in this study.

The three types of optimization problems: size, shape and topology; are considered more in an integrate application. Even this can cause some difficulties in the first phases, but when understanding the way to apply the algorithm, it can lead to better results.

Some algorithms are more efficient and robust than others. Recently introduced algorithms, and old improved ones have reported enhanced results. The optimization of skeletal structures, was demonstrated to be achieved better with: Simulated Annealing (SA), Evolution Strategies (ES), Fully Constrained Design (FCD), Enhanced Genetic Algorithm (GA), Enhanced Honey Bee Mate Optimization (EHBMO), Guided Stochastic Search (GSS) etc.

It was observed that ad hoc algorithm built for specific structures, offer the best results. So for example, the (GSS), point 3.2.4, built for steel truss structures, showed a greater efficiency and robustness compared to other algorithms.

There are only a few real world applications of size structural optimization, since these algorithms are known priory in academic contexts and only in very few specialized companies. There is a lack between the scientific community studies and the real world applications in the construction industry. It is necessary to own a deep knowledge of structural mechanics and numerical methods, in order to apply optimization techniques to real world problems.

Vast applications in the construction industry may bring to a new way of designing, with more efficient structures, versus a sustainable use of resources and less costs.

Further research is necessary to have a more comprehensive detailed state of the art of the structural size optimization algorithms.

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